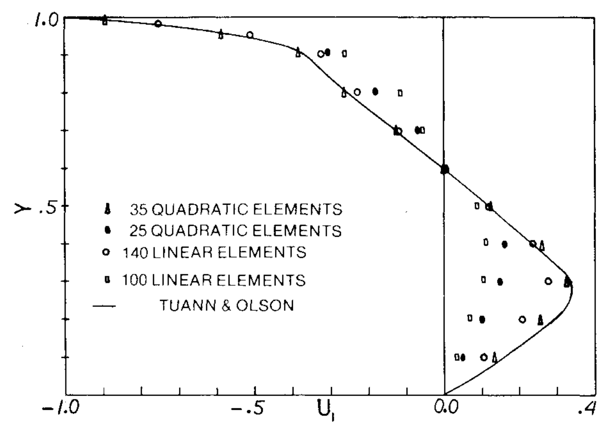

Fig. 1 Horizontal velocity profiles along $x_1 = 0.5$ for Reynolds number 400; bilinear and biquadratic elements.

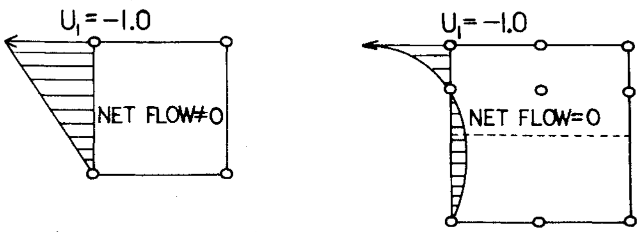

Fig. 2 Mass exchange in a bilinear element (left) at the top left corner (0,1). No net flow n biquadratic elements (right) achieved by shifting of the midnodes.

shown through the driven cavity flow problem. This is solved over the domain $\Omega = [0,1] \times [0,1]$ with boundary conditions $u_1 = u_2 = 0$ on Γ, except along $x_2 = 1$, $0 \leq x_1 \leq 1$, where $u_1 = -1$. Figure 1 shows the horizontal velocity profiles u_1 along the midplane $x_1 = 0.5$ for Reynolds number 400 obtained with bilinear and biquadratic elements in an 11×11 regular mesh, compared with that given by Tuann and Olson.[12] Although biquadratic elements are more accurate than bilinear, both solutions are poor. This is due to the singularities in the horizontal velocity component u_1 at the top corners, which allow a mass exchange to take place within Ω, with the consequent mass imbalance across any internal line $x_1 = a$. This flux is given by $\Delta x_2/2$, where x_2 is the (nondimensional) length of the corner elements in the x_2 direction. The situation is illustrated in Fig. 2. It is clear that the only possible way‡ to avoid this with bilinear elements is the use of a top row of elements thin enough to make the amount of mass exchanged not significant. On the other hand, biquadratic elements provide us with the possibility of shifting the midnodes to balance the flow across the element and provide accurate solutions even for coarse meshes. In Fig. 1 we show two more solutions obtained in a 15×11 mesh obtained by addition of nodes at $x_2 = 0.85$, 0.93, 0.96, and 0.98. The better accuracy of biquadratic elements is significant. Solutions with and without shifting the midnodes at the top were computed, but only the first is shown for clarity.

Conclusions

The use of biquadratic elements with the penalty method not only provides better accuracy but opens a wider range of possibilities to attack pathological situations such as arise in the driven cavity flow. The penalty method provides excellent mass conservation as evidenced by numerical experiments where it can be controlled to within the machine capacity. Further evidence is given by the fact that stream function values calculated by integration of computed velocity field are virtually path independent.[13]

‡The singularity can be eliminated by setting $u_1 = 0$ at the corners, but this fails if bilinear elements are used. Corner elements have only one free node to balance the flow and force the appearance of a checkerboard mode. Quadratic elements can, however, be used.

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Singularities in Unsteady Boundary Layers

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I. Introduction

SINGULARITIES of various kinds are known, or feared, to occur in solutions of the thin-shear-layer ("boundary-layer") equations. The example most commonly quoted is the Goldstein[1] "square root" singularity, in which surface shear

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stress approaches zero, at the point $x=x_0$ say, as $(x_0-x)^{1/2}$. It will be seen below that all of the singularities so far reported involve violation of the principle of conservation of V-component momentum. This principle is of course not enforced by the thin-shear-layer equations, in which the V-component equation of motion is reduced to $\partial p/\partial y=0$. It therefore seems obvious, although the point is apparently not universally accepted, that singularities of the above type will not occur in solutions of the Navier-Stokes equations (or indeed even in solutions of simplified equations in which some better approximate form of the V-component equation is retained). The only kind of singular behavior that undoubtedly occurs in Navier-Stokes solutions is infinite streamline curvature adjacent to a discontinuity in boundary slope, and this scarcely merits further discussion since no unphysical behavior is implied. Therefore, singularities are a result—admittedly a spectacular one—of inaccuracy in the thin-shear-layer approximation, and any attempt to discuss their occurrence in physical (i.e., Navier-Stokes) terms is bound to fail. The justification for studying singularities is that there are many problems where the thin-shear-layer equations are a great simplification and are acceptably accurate everywhere except near a singularity. Much computational expense could be avoided if one could justify neglecting or suppressing the singularity, or treating the neighborhood of the singularity by a Navier-Stokes solution matched to adjoining thin-shear-layer solutions.

The basic theory of singularities of the Goldstein square-root type was extended to unsteady flow by Moore[2] and independently by Rott[3] and Sears[4]; since that time there have been several reviews of laminar separation in general (e.g., Refs. 5 and 6) and of the unsteady problem in particular. Riley[7] argues that unsteady separation is not inevitably accompanied by a singularity; analysis and numerical calculation by Sears' group (e.g., Ref. 8) suggest that a singularity always occurs, within the fluid. The present short Note extends and supersedes Ref. 9.

II. Types of Singularity

Analysis and discussion of the Goldstein square-root singularity, either at the surface or within the stream, will be postponed to Sec. III. Here we discuss other effects which are rightly or wrongly called "singularities."

It is helpful to distinguish the point at which the surface shear stress (or a given component of surface shear stress in a three-dimensional flow) goes to zero, and the points at which the rate of growth of shear layer thickness becomes large. The latter point is of course ill-defined unless the word "large" is arbitrarily quantified; we will merely require $d\delta/dx$ to be of order unity. The point of zero surface shear stress and of large $d\delta/dx$ roughly coincide in two-dimensional steady separation, but in other cases they may not coincide, one may occur without the other, and the use of the word "separation" to describe either effect leads to confusion; theoreticians prefer to reserve the word "separation" for large $d\delta/dx$, but even this is misleading in some cases. It is even more misleading to regard large $d\delta/dx$ as an example of singular behavior; infinite $d\delta/dx$, and even infinite $d^2\delta/dx^2$, can be so regarded, but although large but finite $d\delta/dx$ implies inaccuracy of the thin-shear-layer approximation this is not a matter that affects the mathematical status of the thin-shear-layer equations. It is clear that, in steady flows at least, infinite $d\delta/dx$ will usually imply infinite V, while infinite $d^2\delta/dx^2$ implies infinite $\partial V/\partial x$ at $y=\delta$, and both therefore imply violation of the V-component momentum equation. However, there are several kinds of unsteady flow over bodies of finite length in which large $d\delta/dx$ is calculated numerically or observed experimentally. Experimental values of $d\delta/dx$ could become infinite if a strong transverse vortex developed, as it does in the flow over a pitching airfoil (the line $y=\delta(x)$ might resemble the profile of a wave on the point of breaking). This does not imply infinite V, of course, and the phenomenon might be qualitatively reproducible by boundary-layer calculations without the appearance of any kind of singularity, although in principle the reversed flow near the surface associated with the real vortex could lead to a singularity in that region.

The preceding discussion refers to nearly two-dimensional flow in which the singularity, proper or improper, occurs above a nearly straight line. Bodonyi and Stewartson[10] discuss a singularity occuring above a point, on the axis of the flow over a rotating disk in a counter-rotating fluid. Here, all three velocity components become infinite on the axis, at a finite time after the flow is started. The thin-shear-layer approximation leaves the axial component of velocity, V say, unconstrained, except by the continuity equation. The reason for the appearance of infinite circumferential and radial velocities, in spite of the conservation of momentum in these directions, is that large axial velocity leads to large radial (inward) velocity and thus, via conservation of angular momentum, to large circumferential velocity; this in turn leads to large radial pressure gradients and a runaway to infinite V, etc. (Recall that the thin-shear-layer approximation permits viscous transport of momentum only in the axial direction). The infinite circumferential and radial velocities appear only in an infinitesimal cylinder along the axis so the total momentum or angular momentum varies smoothly with time. Bodonyi and Stewartson's configuration is rather specialized, but similar results may appear in other axisymmetric flows in which the thin-shear-layer approximation implies the abandonment of axial momentum conservation. The effect would not occur in the more normal application of the thin-shear-layer approximation to axisymmetric flow, in which it is radial momentum that is not conserved.

III. The Square-Root Singularity

Moore[2] points out that a prescribed adverse pressure gradient ($dp/dx>0$) reduces the magnitude of the U-component velocity, independent of the sign of the latter; therefore if U is greater than zero on a given streamline, the thin-shear-layer equations will not allow U to become negative under the influence of pressure gradient alone, even in unsteady flow (upstream motion of a solid wall can of course produce negative U by viscous transfer of momentum). Moore says "accordingly, if velocity and shear come to zero they must do so with a singularity in x." If the words "in general" are inserted after "must," this represents the consensus view in steady flow. Moore attributes regular behavior in experimentally observed separation to the influence of the term $\nu\partial^2U/\partial x^2$, neglected in the thin-shear-layer equations, but we shall see that this is not the true explanation. Stewartson[11] has used a triple-deck simplification of the Navier-Stokes equation to examine the inevitability of the square-root singularity in steady flow. His expansions lead to the neglect of $\partial p/\partial y$ in the lower deck and the main deck (which together constitute the shear layer) and he concludes that the singularity still occurs even though the approximations used are more refined than those that yield the thin-shear-layer equations.

The neatest way of demonstrating the possibility of a singularity at the point where $\tau_w=0$ is to differentiate the boundary-layer momentum equation,

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 U}{\partial y^2} \tag{1}$$

twice with respect to y, and put $y=0$; after some use of the continuity equation and the no-slip condition at the surface, we obtain

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 U}{\partial y^2}\right)_w+\frac{\partial}{\partial x}\left\{\frac{1}{2}\left(\frac{\partial U}{\partial y}\right)_w^2\right\}=\nu\left(\frac{\partial^4 U}{\partial y^4}\right)_w \tag{2}$$

Now applying Eq. (1) at the wall (where its left-hand side is zero) and substituting in Eq. (2) gives

$$\frac{\partial}{\partial x}\left\{\frac{1}{2}\left(\frac{\partial U}{\partial y}\right)_w^2\right\}=\nu\left(\frac{\partial^4 U}{\partial y^4}\right)_w-\frac{1}{\mu}\frac{\partial^2 p}{\partial x\partial t} \tag{3}$$

and since p is a known function of x and t (according to the boundary-layer approximation $\partial p/\partial y=0$) we see that the last term in Eq. (3) is known. Integrating Eq. (3) with respect to x near the point of zero shear stress $x=x_s$, we get

$$\left(\frac{\partial U}{\partial y}\right)_w=(x_s-x)^{1/2}\left\{2\left[\frac{1}{\mu}\frac{\partial^2 p}{\partial x\partial t}-\nu\left(\frac{\partial^4 U}{\partial y^4}\right)_w\right]\right\}^{1/2}$$

$$\equiv (x_s-x)^{1/2}C \text{ say} \tag{4}$$

where C is a function of x and t, so that unless C passes through zero at $x=x_s$, $(\partial U/\partial y)_w\sim(x_s-x)^{1/2}$. Explicit integration of the last term in Eq. (3) complicates Eq. (4) but does not alter the conclusion.

Note that if the longitudinal diffusion term $\nu\partial^2U/\partial x^2$ is retained in Eq. (1), then Eqs. (2) and (3) merely acquire an extra term $(1/\rho)\,\mathrm{d}^3p/\mathrm{d}x^3$ which is known and therefore does not materially affect Eq. (4). This refutes Moore's suggestion mentioned earlier.

In steady flow, the singularity does not lead to infinite streamwise acceleration. Using Eq. (4) to give the first term in an expansion for U as a function of y gives

$$U=(x_s-x)^{1/2}Cy+\ldots \tag{5}$$

so that

$$\frac{\partial U}{\partial x}=-\frac{1}{2}(x_s-x)^{-1/2}Cy+(x_s-x)^{1/2}\frac{\partial C}{\partial x}y+\ldots \tag{6}$$

and, using Eq. (6) and the continuity equation to give

$$V=\frac{1}{4}(x_s-x)^{-1/2}Cy^2-\frac{1}{2}(x_s-x)^{1/2}\frac{\partial C}{\partial x}y^2+\ldots \tag{7}$$

we obtain

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{C^2y^2}{4}+(x_s-x)C\frac{\partial C}{\partial x}\frac{y^2}{2}+\ldots \tag{8}$$

which is finite if higher-order terms are well behaved. Equation (1) shows that infinite acceleration would require infinite $\partial^2U/\partial y^2$ (a kink in the U profile) but would tend immediately to smooth out the kink; this of course happens at a trailing edge where, if the boundary-layer approximation is used, infinite $\partial^2U/\partial y^2$ is forced, at one point only, by a discontinuity in boundary conditions.

The normal acceleration is

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=\frac{1}{4}(x_s-x)^{-1}c^2y^3+\text{regular terms} \tag{9}$$

which is zero at $y=0$ (an obvious result) but infinite for $y>0$; the thin-shear-layer approximation does not prohibit infinite normal acceleration, whereas the Navier-Stokes equations do.†

In unsteady flow, Eqs. (5) and (6) hold without change but the acceleration now contains an extra term $\partial U/\partial t$, and from Eq. (5) we have

$$\frac{\partial U}{\partial t}=(x_s-x)^{1/2}\frac{\partial C}{\partial t}y+\frac{\frac{1}{2}Cy}{(x_s-x)^{1/2}}\frac{\mathrm{d}x_s}{\mathrm{d}t}+\ldots \tag{10}$$

Therefore, if the point of zero surface shear stress, $x=x_s$, varies in time the acceleration becomes infinite at $x=x_s$, $y>0$ unless C is zero (higher-order terms cannot affect the conclusion in this case). Since infinite longitudinal acceleration is forbidden by the equations of motion, it follows that the solution of the equations must adjust itself to give $C=0$. Clearly this argument breaks down if $\mathrm{d}x_s/\mathrm{d}t=0$. Therefore a singularity may still occur in an oscillating flow when x_s passes through a minimum or a maximum; in such a case the flow near the separation point is effectively steady.

Similar but more refined arguments were used by Moore on the internal square-root singularity in unsteady flow, in which U and $\partial U/\partial y$, measured in axes moving with the singularity, become zero within the fluid; unless the singularity moves with the fluid the longitudinal acceleration becomes infinite, which is impossible. More generally, a free stagnation point (viewed in some system of axes) will appear, and in that case one would expect the behavior of the velocity field to be regular, as in the case of the Kelvin "cat's eye" pattern in hydrodynamic stability. Williams and Johnson[14] have computed some "semisimilar" flows, which can be reduced from (x,y,t) axes to $(\hat{x},y)$ axes where $\hat{x}$ is a combination of x and t, and in these a singularity can move with the fluid for all time. The main question that remains is whether the internal square-root singularity can appear, or persist, in more general flows, and it is not likely to be possible to settle this analytically. Telionis and Tsahalis[8] found evidence for a singularity in the behavior of their numerical calculations for unsteady flow around a circular cylinder but others (e.g., Ref. 15) do not find this kind of breakdown in the cylinder flow. Proof by numerical calculations is difficult; sets of calculations in which breakdown attributable to singularities do not appear prove nothing except that the singularities are not universal; proof that a given breakdown is attributable to a singularity and not to numerical instability or other malfunction is not straightforward. Intuition is an unreliable guide, since any singularity is an artifact of an inadequate system of equations and not a physical phenomenon. However, Moore's work shows that the V-component velocity is infinite at (and above) the internal singularity—in addition to having infinite normal gradient as in the case of the surface singularity (the infinite V at the singularity does not arise from the $V=0$ constraint at the wall, but is proportional to the cube of $\partial U/\partial y$ upstream of the singularity). The impossibility of infinite longitudinal acceleration suggests that this extreme of behavior of V is unlikely to appear, or to be maintained, in a general unsteady flow. It seems more probable that a free stagnation point will move, either in x or in y, and avoid infinite V and infinite longitudinal acceleration by relaxing the singularity.

IV. Conclusions

The square-root singularity in wall stress is found only in steady or quasisteady flow. It appears because the thin-shear-layer equations do not allow the pressure to vary through the shear layer. Other types of singularity or pathological behavior found in shear layer solutions are also attributable to the failure to conserve V-component momentum. In at least one case[8] an initially regular unsteady flow becomes singular after a finite time.

It is easy to show that wall-stress singularities cannot occur in general unsteady flow since infinite longitudinal acceleration would result. The question of the regularity of the flow at a free stagnation point, the more general unsteady flow singularity postulated by Moore,[3] is more difficult to

†"Inverse" calculations[12,13] specifying δ^* constrain V and prevent singularity. Stewartson's retention of $\partial p/\partial y$ in the upper deck alone evidently does not.

settle. Intuition and comparison with the wall-shear singularity suggests that the free-stagnation-point flow will be singular only in special quasisteady cases where the longitudinal acceleration remains finite.

Acknowledgments

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On the Transonic-Dip Mechanism of Flutter of a Sweptback Wing

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Nomenclature

a = distance of elastic axis behind midchord, percent semichord
b = semichord
g = structural damping coefficient
h = vertical displacement of elastic axis
K_α = spring constant of pitching oscillation
K_h = spring constant of bending oscillation
L = lift per unit span
M = Mach number
M_α = pitching moment about elastic axis per unit span
m = mass of airfoil per unit span
$r_{c.g.}$ = nondimensional radius of gyration about center of gravity
V_F = flutter velocity
$x_{c.g.}$ = distance of center of gravity behind midchord, percent semichord
x_{pv} = distance of pivotal point behind midchord, percent semichord
α = pitching displacement
μ = mass ratio, $m/\pi\rho b^2$
ρ = air density of freestream
$\phi_{h,\alpha}$ = phase difference between bending and torsional oscillation, bending leading the torsional motion
ω_h, ω_α = uncoupled circular frequency of the airfoil in bending and in pitch, respectively
ω_1, ω_2 = first and second natural circular frequency of airfoil
ω_F = circular frequency at flutter

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Introduction

As is well known, the transonic flight region presents critical conditions for the flutter-free requirement of various wing surfaces. This situation is especially severe for sweptback wings since they experience a sharp drop of the flutter boundary (transonic dip).[1,2]

As pointed out by Mykytow,[1] there is a detrimental effect of mass ratio on the flutter boundary in the transonic region; the greater the mass ratio, the deeper the dip. In the experimental data obtained by Farmer and Hanson[2] for a sweptback wing of large aspect ratio, the flutter frequency decreases from 18 Hz at $M = 0.6$ to around 10 Hz near $M = 1.0$, where there is a sharp transonic dip. These characteristics of sweptback wing flutter in the transonic region suggest a mechanism of the transonic dip phenomenon. The purpose of the present study is to investigate this mechanism and identify the essential structural and unsteady aerodynamic features.

Analysis

As pointed out by Cunningham[3] in his paper on pure-bending flutter of a sweptback wing, the first bending mode of the sweptback wing can absorb energy from the airstream. The fundamental mechanism of this can be explained as follows. When we look at the streamwise sections, we notice that they pivot around the axes near or ahead of the leading edge. In this case, the work per cycle done by the aerodynamic pitching moment (about the pivotal point) on the airfoil motion becomes positive if there is a time lag between the airfoil motion and the pitching moment.[4] This destabilizing time lag is caused by the effects of the shed vortices[4] and that of the flow compressibility.[3,5] Since the latter compressibility effect on the time lag is most pronounced in the transonic region, it is quite possible that this mechanism of pure-bending flutter might be dominating the transonic dip phenomenon. The characteristic behavior of sweptback wing flutter such as the strong dependence on the mass ratio and the decrease of flutter frequency in the transonic dip also seem to support this idea. To confirm this, we made some flutter calculations of a two-dimensional wing, which has a similar vibrational characteristic to a streamwise section of a typical sweptback wing.

As already pointed out, the streamwise sections of a sweptback wing in the first natural vibration mode have pivotal points ahead of the leading edge. A close examination of the first natural modes of the flutter models reported in Ref. 6 (the sweptback series), which show a sharp transonic dip of the flutter boundary, has revealed that the pivotal points are located from one to two chords ahead of the